A Story of Volatility Investing and Trading

Part I: Historical Approaches Through Options

Study carried out by the Quantitative Practice
Special thanks to Pierre-Edouard THIERY

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OCTOBRE 2021

Summary

Introduction

Volatility is one of the key parameters in finance: it measures the degree of variation of a trading price series over time. Even though this definition may sound fairly simple at first sight, it is the basement of a broader approach of volatility; indeed this word spans many ideas and theories which play a pivotal role in modern finance. A first way of apprehending volatility is through mathematics: volatility can be computed as the standard deviation of the series of price returns. When computed in such a manner, volatility is said to be "historical", insofar as the computations are based on past data, and so the level of volatility provides us with information about the past behavior of the price series.

However it is also possible to use volatility as a way of gaining insight into the future. This is the concept of implied volatility, whose purpose is precisely to look forward in time. It is based on the idea that the current market prices of derivative instruments mirror the expectations of the market participants regarding what may happen in the future. The simplest approach of implied volatility is premised on European call and put options. For a given strike and a given maturity, the price of the corresponding option is directly observable in the market, and this price contains information about the investors' expectations. Inverting the Black-Scholes formula, we can extract a level of volatility corresponding to the market price: this is the most widespread definition of implied volatility.

Volatility is financial notion that has been known for quite a long time, at least several centuries [1]. Therefore the idea of volatility investing and trading is nothing new: the volatility of an asset may be expected to go up or down in the coming days, weeks or months. Nonetheless, inasmuch as volatility was not a directly "tradable" financial asset, trading volatility long required the use of other instruments. However those approaches were not short of disadvantages; this reality led to the rise of specific instruments, known as "volatility products", providing investors with pure volatility exposure. The most widespread of those instruments is the variance swap. Then new products have also appeared, in order to suit evermore detailed investors' needs.

The purpose is our work is to review the various aspects of volatility investing and trading, from the use of European options to ever more complex financial products devised to enable investors to take ever more precise views on volatility. We will set forth the existing methods to trade volatility, as well as the most widespread volatility strategies and the so-called "volatility risk premium".

This work, entitled "A Story of Volatility Investing and Trading", is divided into three papers. The first one presents the historical approaches of volatility investing, based on European options. In the second paper, we will introduce the variance swap and insist on its role in several volatility strategies. The third paper will be dedicated to what is known as "third generation" volatility products, i.e. products whose purpose is to allow asymmetric bets on volatility.

In this first paper, after putting forth the stylized facts of volatility, we explain how options can be used to invest in volatility, and why such approaches have little interest in practical terms.

1 Volatility and Its Characteristics

Before delving into options-based strategies and volatility products, we remind the reader of the main characteristics of volatility. The first of those characteristics is that volatility jumps when markets crash: when markets face difficult times, volatility tends to spike at high levels. Figure 1 exemplifies this stylized fact of volatility, displaying the moves of S&P 100 implied volatility when market-stressing events occur.

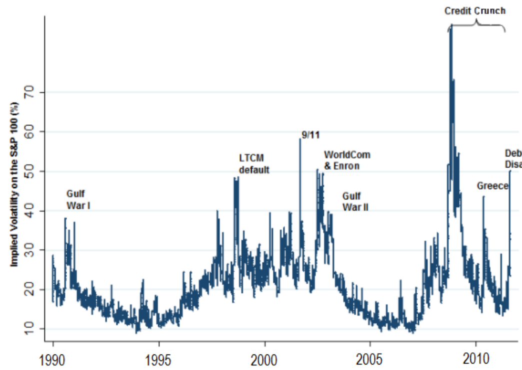

Figure 1: Volatility jumps when markets face difficult times

The second stylized fact is closely related to the first one: when considering a given asset, the volatility is usually negatively correlated to the asset returns. Figure 2 displays the correlation, measured over six weeks, between the S&P500 daily returns and the VIX, which is an index which measures the implied volatility of the S&P500. Figure 3 shows the movements of both the S&P 500 and the VIX index: it is fairly obvious that the two are negatively correlated.

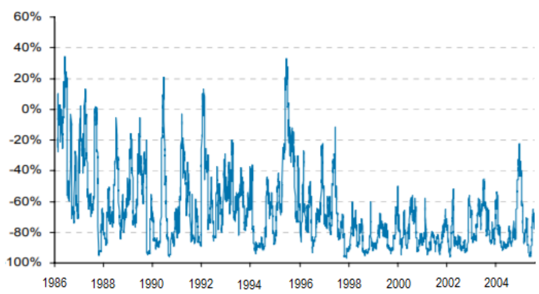

Figure 2: Six-week correlation between changes in S&P500 daily returns and VIX

Figure 3: Comparison of S&P 500 and VIX movements

Two other stylized facts must be mentioned regarding volatility. Contrary to stock returns, volatility displays some mean-reversion: it tends to revert back towards its average value. Besides, volatility experiences high and low regimes, meaning that volatility tends to persist either at a high or low level for some time once it reaches a high or low value.

2 The Traditional Approaches Of Volatility Trading

When one tries to trade the volatility of a given asset, the most straightforward idea consists in relying on European options. It is indeed a well-known fact that those instruments exhibit some vega, meaning that the price of those instruments depends on the underlying's volatility. Using the Black-Scholes formula, if we denote C the price of a call option and P the price of a put option, both having the same characteristics (underlying, strike, maturity), we have:

$$\frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} > 0$$

2.1 Straddles and Strangles

Based on this fact, the simplest way of gaining exposure to volatility is to buy or sell straddles on the chosen underlying. A straddle is merely the combination of a call and a put options, with the same characteristics.

Definition 1 (Straddle)
Taking a long/short position in a straddle consists in taking a long/short position in a call option and a long/short position in a put option, both having the same characteristics T (maturity) and K (strike):

$$Straddle_t(T, K) = Call_t(T, K) + Put_t(T, K)$$

The payoff of a straddle is the following:

$$Straddle_T(T, K) = \begin{cases} K - S_T & \text{if } S_T \leq K \\ S_T - K & \text{otherwise} \end{cases}$$

Figure 3 displays the payoff, taking into account the premium paid upfront.

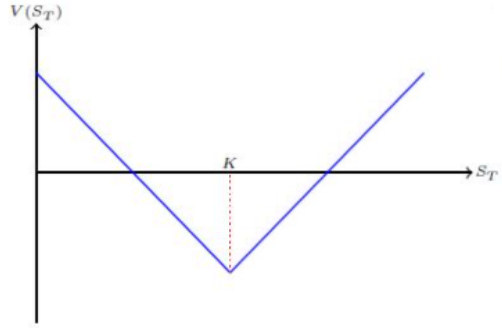

Figure 4: P&L at maturity of a straddle at maturity

There is no difficult in computing the vega of such a product: the vega of straddle is equal to twice the vega of a call option, and so is positive. However, beyond mathematical partial derivatives, looking at the terminal payoff is also instructive: the furthest the terminal price from the strike level, the highest the payoff. If we assume that the price of an asset exhibiting a high level of volatility is not very likely to be equal to the strike at maturity, it means that buying a straddle is a way to make money when volatility is important: it provides investors with a means to take positions on the future realized volatility of an asset.

It is also possible to use strangles instead of straddles: a strangle is made of a put option of strike K_1 and a call option of strike K_2, with $K_1 \leq K_2$. Using strangles allow to reduce the premium which has to be paid upfront: the highest the strike of a call option, the lowest its price; the highest the strike of a put option, the highest its price

$$\frac{\partial C}{\partial K} < 0, \; \frac{\partial P}{\partial K} > 0$$

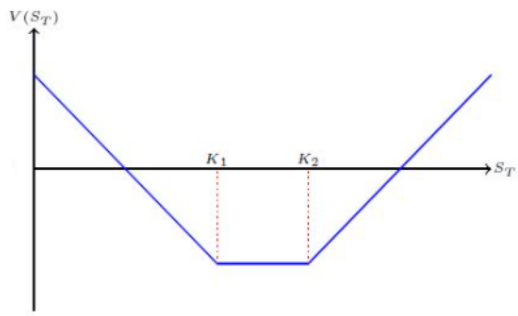

Figure 5: P&L of a strangle at maturity

If a strangle is cheaper than a straddle, it is also less effective in taking a position on volatility insofar as, as shown by the payoff of a strangle (Figure 4, taking into account the premium paid upfront), it requires that the asset price moves sufficiently, i.e. that $S_T > K_2$ or $S_T < K_1$, to receive a positive payoff.

The use of straddles or strangles may be an intuitive way to trade volatility, but it is a rather inefficient one for several reasons.

First, such instruments do not provide a pure exposure to volatility. It is possible to wind up with a terminal payoff being worth zero even though the asset has displayed lots of volatility: all it takes is a terminal price S_T being equal to the strike price K (straddle), or being in the interval $[K_1, K_2]$ (strangle).

Second, a position in straddles or strangles is no longer delta neutral. Those instruments are sensitive to price movements, and thus are not pure volatility products.

2.2 Delta-hedging of an Option

An intuitive way of circumventing this second weakness is to consider delta-hedged options. As mentioned earlier, a European option price depends on the underlying's volatility, and the delta-hedging allows the investor to have a zero exposure to the moves of the asset price. The strategy is the following:

- We sell an option, be it a call or a put, with maturity T, strike K, using an implied volatility σ_i to price the product; its price is denoted $V(S_t, t, \sigma_i)$.
- We delta-hedge our position, buying δ_t stocks at time t, where $\Delta_t = \frac{\partial}{\partial S} V(S_t, t, \sigma)$; we assume that the hedge is made using a constant implied volatility denoted σ_h.

Based on this scheme, it is possible to compute the overall $P\&L$ of the strategy.

Proposition 1 ($P\&L$ of a delta-hedged option)
At maturity T, and if we assume that the dynamics of the underlying S is given by:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t$$

the P&L of the delta-hedged strategy is:

$$P\&L_T = e^{-rT}\left[-V(S_0, 0, \sigma_h) + V(S_0, 0, \sigma_i)\right]$$

$$+ \int_0^T e^{r(T-t)}(-\sigma_t^2 + \sigma_h^2)\frac{S_t^2}{2}\partial_{xx} V(S_t, t, \sigma_h)dt$$

where r is the risk-free rate, σ_i the implied volatility used to price the option sold at t = 0, σ_h the implied volatility used when carrying out the hedge, σ_t the instant volatility of S, and where ∂_x denotes the first partial derivative of a function regarding the spot value S_t.

The proof of this formula is interesting [2] [3] as it dwells on the financial realities which lie behind such a strategy. Indeed, to find the expression mentioned in Proposition 1, we write the $P\&L$ as the sum of three terms: the value of underlying shares owned at maturity, minus the total debt at maturity and the payoff paid at maturity.

First, it is important to remind the reader of the Black-Scholes stochastic derivatives equation: for any volatility σ

$$\partial_t V(S, t, \sigma) + rS\partial_x V(S, t, \sigma) + \frac{\sigma^2 S^2}{2}\partial_{xx} V(S, T, \sigma) - rV(S, t, \sigma) = 0$$

and $V(S, T, \sigma) = f(S)$ where f is the payoff function.

At $t = 0$, the trader sells the option priced thanks to the Black-Scholes formula with an implied volatility being equal to σ_i: he or she is paid $V(S_0, 0, \sigma_i)$. In order to start the hedging of the option, the trader must buy $N_0 = \partial_x V(S_0, 0, \sigma_h)$ stocks, where $N_t = \partial_x V(S_t, t, \sigma_h)$ now denotes the amount of stocks owned by the trader at t. To do so, he or she borrows at $t = 0$:

$$\partial_x V(S_0, 0, \sigma_h)S_0 - V(S_0, 0, \sigma_i)$$

If we denote β the cumulative debt, we then have:

$$\beta_0 = \partial_x V(S_0, 0, \sigma_h)S_0 - V(S_0, 0, \sigma_i)$$

The dynamics of β_t is given by a simple financial reasoning. If we consider what happens between t and $t+\Delta t$, we can write:

$$\beta_{t+\Delta t} = \beta_t + \underbrace{r\beta_t\Delta_t}_{A} - \underbrace{N_t S_{t+\Delta_t}}_{B} + \underbrace{N_{t+\Delta t}S_{t+\Delta t}}_{C}$$

A corresponds to the interest which has to be paid on the amount of debt β_t between t and $t+\Delta t$. B is the current wealth of our stock positions. C is the amount of money we have to borrow in order to update our stock positions. If we assume that $\Delta t \to 0$, we can write:

$$d\beta_t = r\beta_t dt + (S_t + dS_t)dN_t$$

$$= d(N_t S_t) - N_t dS_t + r\beta_t dt$$

We then apply the Ito formula to $V(S_t, t, \sigma_h)$:

$$dV(S_t, t, \sigma_h) = \partial_t V(S_t, t, \sigma_h)dt + \partial_x V(S_t, t, \sigma_h)dS_t$$

$$+\frac{1}{2}\sigma_t^2 S_t^2 \partial_x x V(S_t, t, \sigma_h)dt$$

Using the Black-Scholes EDS for V, we can rewrite:

$$dV(S_t, t, \sigma_h) =$$

$$\Big[rV(S_t, t, \sigma_H) - rS_t\partial_x V(S_t, t, \sigma_h)$$

$$-\frac{\sigma_h^2 S_t^2}{2}\partial_{xx} V(S_t, t, \sigma_h) + \frac{\sigma_t^2 S_t^2}{2}\partial_{xx} V(S_t, t, \sigma_h)\Big]dt$$

$$+\partial_x V(S_t, t, \sigma_h)dS_t$$

$$= (\sigma_t^2 - \sigma_h^2)\frac{S_t^2}{2}\partial_{xx} V(S_t, t, \sigma_h)dt + r\Big[V(S_t, t, \sigma_h) - S_t\partial_x V(S_t, t, \sigma_h)\Big]dt$$

$$+ \underbrace{\partial_x V(S_t, t, \sigma_h)dS_t}_{N_t dS_t}$$

We can now replace in the expression of $d\beta_t$ both $N_t S_t$ and $N_t dS_t$:

$$d\beta_t - r\beta_t dt = d\left[\underbrace{S_t\partial_x V(S_t, t, \sigma_h)}_{\alpha_t}\right] - r\alpha_t dt$$

$$-dV(S_t, t, \sigma_h) + rV(S_t, t, \sigma_h)dt + (\sigma_t^2 - \sigma_h^2)\frac{S_t^2}{2}\partial_{xx} V(S_t, t, \sigma_h)dt$$

We know that $d(e^{-st}\gamma_s) = e^{-rs}d\gamma_s - re^{-rs}\gamma_s ds = e^{-rs}(d\gamma_s - r\gamma_s ds)$. So if we multiply the above equation by e^{-rs} and then we integrate from 0 to t, we get:

$$e^{-rt}\beta_t - \beta_0 = e^{-rt}\alpha_t - \alpha_0 - e^{-rt}V(S_t, t, \sigma_h) + V(S_0, 0, \sigma_h) + \int_0^t e^{-rs}(\sigma_s^2 - \sigma_h^2)\frac{S_s^2}{2}\partial_{xx}V(S_s, s, \sigma_h)ds$$

We then consider the equation with $t = T$ and we multiply it by e^{rT} in order to get β_T. After a few simplifications, we get:

$$\beta_T = e^{rT}\left[V(S_0, 0, \sigma_h) - V(S_0, 0, \sigma_i)\right] + S_T\partial_x V(S_T, T, \sigma_h) - V(S_T, T, \sigma_h) + \int_0^T e^{-r(T-t)}(\sigma_t^2 - \sigma_h^2)\frac{S_t^2}{2}\partial_{xx}V(S_t, t, \sigma_h)dt$$

We now know the total debt at maturity. This quantity is necessary to compute the $P\&L$ at maturity since:

$$P\&L_T = N_T S_T - \beta_T - f(S_T)$$

and so we find:

$$P\&L_T = e^{-rT}\left[-V(S_0, 0, \sigma_h) + V(S_0, 0, \sigma_i)\right] + \int_0^T e^{r(T-t)}(-\sigma_t^2 + \sigma_h^2)\frac{S_t^2}{2}\partial_{xx}V(S_t, t, \sigma_h)dt$$

From this expression, it is possible to go further in order to approximate the $P\&L$ at maturity. To do so, we see V as a function of σ^2, and we linearize it around σ_h^2: $W(S_t, t, v = \sigma^2) = V(S_t, t, \sigma)$. So we have:

$$-V(S_0, 0, \sigma_h) + V(S_0, 0, \sigma_i) = -W(S_0, 0, v_h) + W(S_0, 0, v_i) = (v_i - v_h)\partial_v W(S_0, 0, v_i)$$

However, if $f(x) = g(x^2)$, then $g'(x^2) = \frac{f'(x)}{2x}$, thus

$$\partial_v W(S_0, 0, v_h) = \frac{\partial_\sigma V(S_0, 0, \sigma_h)}{2\sigma_h}$$

Besides we know that the vega of a call/put option can be rewritten

$$\nu = x^2(T - t)\sigma\gamma_{call}$$

So in our case,

$$-V(S_0, 0, \sigma_h) + V(S_0, 0, \sigma_i) = (\sigma_i^2 - \sigma_h^2)\frac{S_0^2 T}{2}\gamma_{call}$$

Therefore, the $P\&L$ can be rewritten:

$$P\&L_T = Te^{rT}(\sigma_i^2 - \sigma_h^2)\frac{S_0^2}{2}\partial_{xx}V(S_0, 0, \sigma_h) + \int_0^T e^{r(T-t)}(\sigma_h^2 - \sigma_t^2)\frac{S_t^2}{2}\partial_x xV(S_t, t, \sigma_h)dt$$

The final step consists in introducing $\hat{\sigma}^2 = \frac{1}{T}\int_0^t \sigma_t^2 dt$ in the equation:

$$P\&L_T = \underbrace{(\sigma_i^2 - \hat{\sigma}^2)Te^{rT}\frac{S_0^2}{2}\partial_{xx}V(S_0, 0, \sigma_h)}_{A} + \underbrace{(\hat{\sigma}^2 - \sigma_h^2)T\left\{\frac{e^{rT}S_0^2}{2}\partial_{xx}V(S_0, 0, \sigma_h) - \frac{\int_0^T e^{r(T-t)}\frac{S_t^2}{2}\partial_{xx}V(S_t, t, \sigma_h)}{T}\right.}_{B} + \underbrace{\int_0^T e^{r(T-t)}(\hat{\sigma}^2 - \sigma_t^2)\frac{S_t^2}{2}\partial_{xx}V(S_t, t, \sigma_h)dt}_{C}$$

Thanks to this expression, we now understand what the risks of trading volatility through a delta-hedged option are. A corresponds to a variance risk, and, as we will see in our second paper, it amounts to a variance swap exposure, exchanging the initial implied volatility against the realized volatility $\hat{\sigma}^2$ on the period $[0, T]$. B corresponds to a vega risk: the option is hedged with a given volatility σ_h, but this volatility is not necessarily equal to the realized volatility. This term would be null if the trader were able to precisely hedge his or her option at this level, which is however unknown before T. Last but nor least, C corresponds to a path dependency risk: the path of the instantaneous volatility σ_t leads to different values for $\sigma_t^2 - \hat{\sigma}^2$.

The only risk which may be caused by a true exposure to volatility is the first one, A. Studies have shown that the variance risk is responsible for only half of the total $P\&L$ when hedging an option [4]. This proves that delta-hedging options is not a way of gaining even a satisfactory exposure to volatility. Furthermore, other reasons explain why delta-hedging options is not the practical solution for trading volatility: we have not taken into account dividends in our model; rates were assumed to be constant; transaction costs were ignored.

When it comes to volatility trading, the shortcomings of those two intuitive approaches are the reason why new derivatives instruments have emerged in financial markets, whose purpose is to gain pure exposure to volatility without having to bear all the other risks.

Conclusion

In the first of our three papers on volatility investing and trading, we have dwelled on the option-based intuitive approaches. Indeed options exhibit some dependency on the volatility of the underlying, so using options sounds very natural when it comes to volatility investing. This can be done either by buying/selling straddles; a cheaper, but also less effective, version of this strategy can be implemented through strangles.

However there are some shortcomings in using straddles or strangles. First, there is no guarantee that a high level of volatility before the maturity of the strategy will deliver a positive payoff. Second, such strategies are not delta-neutral: the exposure to volatility is not pure, insofar as investors are also exposed to the moves of the underlying price.

A simplest solution exists to circumvent this second weakness: delta-hedging. Nonetheless, the mathematical expression of a delta-hedged option P&L shows that, once again, the investor faces other risk than the volatility one. Statistical studies have even proved that the volatility risk when delta-hedging an option accounts for only half of the overall P&L. Furthermore, our analysis has not taken into account all the aspects of a practical implementation of such a strategy, such as transaction costs, variable rates of dividends.

So the interest of those traditional approaches is now mainly historical and academic. Indeed, in order to enable investors to properly trade volatility without facing undesired risks, new products have emerged over the past decades. The most simple, and most widely used, of them is the so-called variance swap, where a predetermined level of volatility is exchanged against the realized volatility at maturity. This will be the focus of our second paper.

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A propos d'Awalee

Cabinet de conseil indépendant spécialiste du secteur de la Finance.

Nous sommes nés en 2009 en pleine crise financière. Cette période complexe nous a conduits à une conclusion simple : face aux exigences accrues et à la nécessité de faire preuve de souplesse, nous nous devions d'aider nos clients à se concentrer sur l'essentiel, à savoir leur performance.

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