# AWALEE NOTES

## Copula in dependence modeling and risk measure estimating for cross-asset portfolio

### Part I : Model and Estimation

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Summary: In this note (Part I), we introduce the GARCH-Copula framework in the modeling of market returns with a special focus on extreme co-movements. In an empirical study with different asset classes (Equities, Bonds, Commodities and Foreigner Exchanges) over the past ten years, we investigate the interdependencies between asset classes and between regions. The estimation results show that the diversification benefits in multi-asset portfolio could be seriously diminished under market turbulence, due to the intensified cross-asset and cross-regional dependences.

## 1. INTRODUCTION

After the major financial crisis of 2008, the interdependence between different asset markets has gained more and more interest among market participants, policy makers and academics. The diversification benefits of multi-asset portfolios can be seriously reduced by an increase in cross asset correlation. Financial stability could also be deteriorated by cross market contagions.

The correlation between market returns is observed to be higher in case of extreme events. The commonly used correlation coefficient in the Gaussian framework could be less eligible in such case (see e.g. Poon, et al., (2004)). Academics and practitioners begin to use copula to model complex dependence structure between assets. Jondeau & Rockinger (2006) studied four major stock markets in the GARCH-Copula approach while Aloui, et al., (2011) use a similar methodology to study emerging stock markets during market panics.

The GARCH-Copula framework has the flexibility of modeling separately the marginal distribution and the dependence structure. The GARCH model along with fat tailed distribution allows us to filter serial correlation and to specify non-normal distributions.

The large availability of copula specifications gives us the possibility to model non-linear dependencies. These features make this approach particularly suitable for market return modeling in financial turbulence.

In this note, we briefly introduce the framework in the section 2 and its estimation process in the section 3. The section 4 is dedicated to an empirical study with different asset markets. Section 5 gives some concluding remarks.

## 2. COPULA IN DEPENDENCE MODELING

In this section, we give in the first place the definition of the copula and its characteristics. Since the extreme co-movement issue is our priority concern in this note, we introduce in the second place the notions of tail dependences.

To give a more concrete idea, we give in the end of this section some examples of copula and their specificities.

### a. Definition

A copula is a function that links univariate margins to their multivariate distribution. More formally, an n-dimensional copula $C$ is defined as the joint cumulative distribution with uniform marginal distributions $(U_i)$ defined on $[0,1]$.

$$C(u_1, \ldots, u_i, \ldots, u_n) = Pr\left\{U_1 < u_1, \ldots, U_i < u_i, \ldots, U_n < u_n\right\}$$

Sklar's theorem outlines the relationship between a joint distribution and a copula. Let F be the n-dimensional distribution with margins $F_i$. Then there exists a copula such that

$$F(X_1, \ldots, X_i, \ldots, X_n) = C\left(F_1(X_1), \ldots, F_i(X_i), \ldots, F_n(X_n)\right)$$

This theorem allows us to study separately the dependence structure of multivariate distributions and the marginal distributions. Indeed, one can identify in the first place the marginal distributions and then specify the appropriate copula to describe the dependence structure. This approach is in particular suitable even when the multivariate normality cannot be applied.

For the marginal distribution modeling, many approaches are available. A Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) model is used here as a filter to remove any serial dependency from the returns. For the dependence structure, using the copula has the advantage of capturing the dependence in the tails of the distribution.

### b. Tail dependence

The coefficient of tail dependence measures the probability that two random variables simultaneously take extreme values. More formally, given two random variables $X_1$ and $X_2$ with marginal distribution functions $F_1$ and $F_2$, the coefficient of lower tail dependence is defined as:

$$\lambda_L = \lim_{u \mapsto 0^+} P\left\{X_1 \leq F_1^{-1}(u) \mid X_2 \leq F_2^{-1}(u)\right\}$$

which measures the probability that a variable $X_1$ takes an extreme low value (not larger than the $u$-quantile of the margin) given that an extreme low value is already observed for the other variable $X_2$.

Similarly, the upper tail dependence measures the probability of concurrently observing large values for the two variables and is defined as:

$$\lambda_U = \lim_{u \mapsto 1^-} P\left\{X_1 > F_1^{-1}(u) \mid X_2 > F_2^{-1}(u)\right\}$$

In the copula framework, the coefficients of tail dependence can be written as follows. Given that *C* is the copula for the two variables, we have

$$\lambda_L = \lim_{u \mapsto 0^+} \frac{C(u,u)}{u}$$
$$\lambda_U = \lim_{u \mapsto 1^-} \frac{1 - 2u + C(u,u)}{u}$$

This gives us a simple way to compute the tail dependence coefficients.

### c. Some examples of copula

In the following, we introduce some copulas that will be used in our empirical study. We choose three typical copula functions (Gaussian, Student-t and Clayton), which are also the most frequently referenced, in order to show the flexibilities of the dependence modeling in the copula framework.

More specifically, one can note from this section that different characteristics in terms of extreme co-movement could be represented in various copula specifications.

#### i. Gaussian copula

The cumulative distribution function (cdf) of a Gaussian copula is defined by:

$$C(u_1, u_2; \rho) = \Phi_R\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right) \tag{1}$$

where $\Phi_R$ is the joint cdf of a multivariate standard normal distribution with $\rho \in$ *[-1,1]* the correlation parameter. The density of a 2-dimensional Gaussian copula is defined as follows:

$$c(u_1, u_2; \rho) =$$
$$\frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\begin{pmatrix}\Phi^{-1}(u_1)\\ \Phi^{-1}(u_2)\end{pmatrix}' (R^{-1} - I)\begin{pmatrix}\Phi^{-1}(u_1)\\ \Phi^{-1}(u_2)\end{pmatrix}\right)$$

where $R$ is the (2,2) dimension correlation matrix with $\rho$ the correlation parameter, $I$ is the identity matrix and $\Phi^{-1}$ the inverse cdf of a standard normal.

The Gaussian copula does not allow joint extreme events, therefore, it cannot measure tail dependence.

#### ii. Student-t copula

Similarly, the cdf of a 2-dimensional Student-t copula is defined by:

$$C(u_1, u_2; \rho, \upsilon) = T_{R,\upsilon}(T_v^{-1}(u_1), T_\upsilon^{-1}(u_2)) \tag{2}$$

where $T_\nu$ is the univariate Student-t cdf with degree-of-freedom parameter $\nu \in$ *[2,∞[* and $T_{R,\nu}$ is the bivariate Student-t cdf having correlation matrix R (with $\rho \in$ *[-1,1]* the correlation parameter) and degree-of-freedom $\nu$. The corresponding Student-t copula density is:

$$c(u_1, u_2; \rho, \upsilon) =$$
$$\frac{1}{\sqrt{1-\rho^2}} \frac{\Gamma\left(\frac{\upsilon+2}{2}\right)\Gamma\left(\frac{\upsilon}{2}\right)\left(1 + \frac{1}{\upsilon}\begin{pmatrix}T_\nu^{-1}(u_1)\\ T_\nu^{-1}(u_2)\end{pmatrix}' R^{-1}\begin{pmatrix}T_\nu^{-1}(u_1)\\ T_\nu^{-1}(u_2)\end{pmatrix}\right)^{-\frac{\upsilon+2}{2}}}{\left(\Gamma\left(\frac{\upsilon+2}{2}\right)\right)^2 \prod_{i=2}^{2}\left(1 + \frac{1}{\upsilon}(T_\upsilon^{-1}(u_i))^2\right)^{-\frac{\upsilon+1}{2}}}$$

where $\Gamma$ is the gamma function.

Compared to the Gaussian copula, the Student-t copula introduces an additional parameter which is the degree-of-freedom $\nu$. A larger value of $\nu$ corresponds to a smaller extreme co-movements. The student-t dependence structure allows measuring the symmetric tail dependences and the coefficients of lower and upper tail dependence can be computed from the two copula parameters:

$$\lambda_L = \lambda_U = 2T_{\upsilon+1}\left(-\sqrt{\upsilon+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)$$

where $T_{\nu+1}$ denotes the univariate Student-t cdf with $\nu+1$ degree-of-freedom. Increasing the correlation $\rho$ and decreasing the degrees of freedom $\nu$ enlarge the tail dependences.

#### iii. Clayton copula

The 2-dimensional Clayton copula is defined by:

$$C(u_1, u_2; \delta) = (u_1^{-\delta} + u_2^{-\delta} - 1)^{\frac{1}{\delta}} \tag{3}$$

where $\delta > 0$ is a parameter controlling the dependence. The case $\delta \to \infty$ indicates a perfect dependence while $\delta \to 0$ implies independence. The Clayton copula only has lower tail dependence:

$$\lambda_L = 2^{\frac{1}{\delta}}$$

Accordingly, compared to the Student-t copula, the Clayton copula exhibits asymmetric tail dependence. If one expects a larger dependence in the negative tail than in the positive tail, the Clayton copula could provide a more reasonable fit whereas the Student-t copula with symmetric tail dependence structure could be too restrictive.

## 3. ESTIMATION PROCESS

In order to take into account the dynamics of the return series and detect the complex dependence structure between assets, we employ a combination of GARCH approach and copula model. At the estimation level, we describe in the following the two-steps process. Financial returns generally exhibit features like autocorrelation, heteroscedasticity and fat tail. To consider these features, in the first step, an AR(1)-GARCH(1,1) model with generalized error distribution (GED) residuals are estimated for asset return series $r_t$, as described in the following equations:

$$r_t = \mu + \mathbf{a} r_{t-1} + \epsilon_t \tag{4}$$

$$\epsilon_t = \sigma_t z_t \tag{5}$$

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{6}$$

$$z_t \sim \mathrm{GED}(0, 1, \kappa) \tag{7}$$

Equation (4) gives the dynamics of the return with an AR(1) process, yielding the parameter **a** while $\mu$ is the mean and $\epsilon_t$ is the innovation term. Equation (5) defines the innovation as the product of conditional volatility $\sigma_t$ and the residual term $z_t$. Equation (6) specifies the dynamics of conditional variance with parameters ($\omega$,$\alpha$,$\beta$). Equation (7) shows that the residuals follow a GED with zero mean, unit variance and a shape parameter ($\kappa$) measuring the tail-thickness of the distribution[1]. The parameters are estimated by the maximum likelihood method. Including or not the autoregressive part (AR(1)) in the specification depends on the test results that will be given in the empirical study.

The residuals $z_t$ are then collected for further analysis. By using the empirical distribution functions, all the series of the residuals are converted into uniform variables which will be employed in the dependence structure estimation.

In the second step, a bivariate copula is estimated for each pair of the asset returns. By using the Bayesian Information Criterion (BIC)[2], we choose the best fit from the three copulas described above, namely Gaussian, Student-t and Clayton copulas.

## 4. EMPIRICAL STUDY

We investigate in this section the interdependency between different asset market indices through an empirical study.

### a. Data description

The data consist of five indices including four types of assets (Stocks, Bonds, Commodities and Currency). More specifically, we have two stock market indices representing Developed Market (MSCI World Index) and Emerging Market (MSCI Emerging Market Index), one government bond index (IBOXX Euro Eurozone Sovereigns Bond Index), one commodity index (Bloomberg Commodity Index) and one exchange rate (EUR-USD). The returns are calculated by taking the log difference of closing prices on two consecutive trading days. The sample period covers daily returns over 10 years from 02 October 2006 to 30 September 2016, yielding 2610 observations.

In Table 1, we give some statistical description for all the indices in order to examine the characteristics of their empirical distributions. The statistics show that the two stock markets and the commodity market are more volatile than the bond market and the exchange rate.

Table 1: Descriptive Statistics for Daily Market Returns

| | MSCI World | Euro Sov. Bonds | MSCI Emerging | Commodity | EUR-USD |
|---|---|---|---|---|---|
| Mean | 0.02% | 0.02% | 0.02% | -0.02% | 0.00% |
| Sd | 1.05% | 0.24% | 1.06% | 1.10% | 0.63% |
| Min | -7.15% | -1.21% | -7.75% | -6.40% | -2.43% |
| Max | 8.72% | 1.59% | 8.40% | 5.65% | 3.46% |
| Skew | -0.46 | -0.04 | -0.31 | -0.31 | 0.10 |
| Ex.Kurtosis | 8.59 | 3.18 | 8.52 | 2.98 | 2.04 |
| J-B | 8141.42*** | 1101.65*** | 7957.23*** | 1011.69*** | 460.77*** |
| Q(10) | 47.84*** | 38.17*** | 141.51*** | 12.60 | 8.40 |
| $Q^2(10)$ | 2512.94*** | 227.07*** | 2540.63*** | 783.10*** | 492.39*** |
| ARCH(10) | 820.93*** | 126.86*** | 731.21*** | 346.95*** | 230.92*** |

*Notes: This table reports descriptive statistics for daily log returns of the five indices. The sample covers from 02 October 2006 to 30 September 2016. Jarque-Bera (J-B) statistics test the null hypothesis of normality. Q(10) and $Q^2(10)$ are the Ljung-Box statistics for autocorrelation in returns and squared returns with 10 lags. ARCH(10) is the Lagrange Multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) up to 10 lags. Significance is denoted by super-scripts at the 1% (***), 5% (**), and 10% (*) levels for all tests.*

[1] The standard normal distribution has a parameter equal to 2. For $\kappa < 2$, the distribution has thicker tails than the normal distribution and for $\kappa > 2$, the distribution has thinner tails than the normal distribution.

[2] BIC is a criterion for model selection among a set of model specifications, based on the likelihood function while penalizing the number of parameters.

Table 2 : Correlation matrix

| | MSCI World | Euro Sov. Bonds | MSCI Emerging | Commo | EUR-USD |
|---|---|---|---|---|---|
| MSCI World | 1 | 0.015 | 0.364 | 0.061 | -0.018 |
| Euro Sov. Bonds | | 1 | -0.134 | -0.129 | -0.090 |
| MSCI Emerging | | | 1 | 0.420 | 0.201 |
| Commo | | | | 1 | 0.380 |
| EUR-USD | | | | | 1 |

Four of five indices are negatively skewed while the exchange rate exhibits a positive skewness (see row Skew). All data series have positive excess kurtosis (see row Ex.Kurtosis), which indicates that the empirical distributions exhibit fatter tails than the normal distribution. Therefore, it is not surprising that the Jarque-Bera (J-B) statistics are highly significant for all series, confirming the non-normality of the return distributions.

The Ljung-Box statistics (Q(10)) are significant for the first three indices, suggesting that serial correlations exist in these indices while commodity index and exchange rate do not seem to exhibit autocorrelation effects.

Nevertheless, all the return series display ARCH effects as indicated by the significant Ljung-Box statistics for the squared returns ($Q^2(10)$) and confirmed by the ARCH LM test (ARCH(10)). As suggested by these autocorrelation tests, we specify an AR(1)-GARCH(1,1) model for the first three return series and a GARCH(1,1) model for the last two.

Table 2 displays the unconditional correlations for all series. Relatively larger correlations are found for the following pairs. For the whole sample period, Developed Stock Markets and Emerging Stock Markets are positively related ($\rho = 0.364$), indicating the co-movements between the two stock markets.
Emerging Stock Markets and Commodity Market exhibit strong dependency ($\rho = 0.420$) which could be explained by the narrow relationship between the emerging country activities and the commodity market. Indeed, emerging countries could drive from the demand side (e.g. China) and also in terms of supply (e.g. Brazil) in commodity market. Bond market has rather negative correlations with other markets.

## b. Estimation results

### i. Marginal model estimation results

We first examine the marginal model. As explained before, either an AR(1)-GARCH(1,1) or a GARCH(1,1) specification is fitted to filter serial correlation in each return series.

Table 3: Estimates of the marginal model for all return

| | MSCI World | Euro Sov. Bonds | MSCI Emerging | Commodity | EUR-USD |
|---|---|---|---|---|---|
| Mean equation | | | | | |
| $\mu$ | 6.69E-04 | 2.07E-04 | 4.17E-04 | -6.81E-05 | 4.93E-05 |
| a | 0.121 | 0.050 | 0.189 | - | - |
| | (1.99E-02) *** | (2.04E-02) ** | (1.93E-02) *** | | |
| Conditional variance equation | | | | | |
| $\omega$ | 1.72E-06 | 9.93E-08 | 1.25E-06 | 3.12E-07 | 9.20E-08 |
| | (4.21E-07) *** | (3.51E-08) *** | (3.55E-07) *** | (1.64E-07) * | (4.86E-08) * |
| $\alpha$ | 0.120 | 0.060 | 0.094 | 0.040 | 0.041 |
| | (1.61E-02) *** | (1.15E-02) *** | (1.23E-02) *** | (7.04E-03) *** | (5.67E-03) *** |
| $\beta$ | 0.865 | 0.924 | 0.893 | 0.958 | 0.958 |
| | (1.62E-02) *** | (1.51E-02) *** | (1.32E-02) *** | (7.08E-03) *** | (5.50E-03) *** |
| Shape parameter | | | | | |
| $\kappa$ | 1.243 | 1.387 | 1.482 | 1.422 | 1.480 |
| | (4.71E-02) *** | (5.03E-02) *** | (5.81E-02) *** | (5.58E-02) *** | (5.75E-02) *** |

*Notes: This table reports the results of GARCH estimations. Parameters are given in equation (4), (6), and (7). The values between brackets give the standard error of the estimated parameters. Significance of parameters is denoted by super-scripts at the 1% (***), 5% (**), and 10% (*) levels.*

Table 3 displays the estimates of the marginal model. For the AR process in the first three return series, the estimated parameters a are positive and statistically significant, suggesting that the returns are indeed positively autocorrelated. The returns of day *t-1* do influence the returns of day *t*.

The parameters in the conditional variance equation are significant for all the series. The sum $\alpha+\beta$ is close to one, which indicates a high persistence in variance dynamics, i.e. large (respectively small) movements in the conditional variance are more likely to be followed by large (respectively small) movements.

Table 4: Copula estimation results for the dependence structure

| | | MSCI World | | | |
|---|---|---|---|---|---|
| Euro Sov. Bonds | Family | Gaussian | | | |
| | Par1 | 0.0475 | | | |
| | s.e.1 | (0.0196) | | | |
| | Par2 | - | | | |
| | s.e.2 | - | | | |
| | Kendall's tau | 0.0310 | | | |
| | Lower TD | - | | | |
| | Upper TD | - | | | |
| | | MSCI World | Euro Sov. Bonds | | |
| MSCI Emerging | Family | Gaussian | Student-t | | |
| | Par1 | 0.2632 | -0.0990 | | |
| | s.e.1 | (0.0177) | (0.0210) | | |
| | Par2 | - | 9.2497 | | |
| | s.e.2 | - | (1.9503) | | |
| | Kendall's tau | 0.1718 | -0.0663 | | |
| | Lower TD | - | 0.0052 | | |
| | Upper TD | - | 0.0052 | | |
| | | MSCI World | Euro Sov. Bonds | MSCI Emerging | |
| Commodity | Family | Clayton | Student-t | Student-t | |
| | Par1 | 0.0382 | -0.0869 | 0.3562 | |
| | s.e.1 | (0.0210) | (0.0207) | (0.0173) | |
| | Par2 | - | 12.1222 | 13.8890 | |
| | s.e.2 | - | (3.2310) | (4.3051) | |
| | Kendall's tau | 0.0156 | -0.0546 | 0.2315 | |
| | Lower TD | 0.0000 | 0.0016 | 0.0180 | |
| | Upper TD | - | 0.0016 | 0.0180 | |
| | | MSCI World | Euro Sov. Bonds | MSCI Emerging | Commodity |
| EUR-USD | Family | Student-t | Student-t | Student-t | Gaussian |
| | Par1 | -0.0309 | -0.0573 | 0.1538 | 0.3518 |
| | s.e.1 | (0.0206) | (0.0217) | (0.0205) | (0.0162) |
| | Par2 | 17.7717 | 6.5481 | 10.7017 | - |
| | s.e.2 | (6.7178) | (1.0308) | (2.7164) | - |
| | Kendall's tau | -0.0229 | -0.0338 | 0.0953 | 0.2289 |
| | Lower TD | 0.0003 | 0.0209 | 0.0129 | - |
| | Upper TD | 0.0003 | 0.0209 | 0.0129 | - |

*Notes: This table displays the results of copula estimations. Each pair of indices is fitted by one of the three copulas. Parameters are given in equation (1), (2), and (3) for each family of copula. Par1 is the first copula parameter, which is $\rho$ in the case of Gaussian and Student-t Copula, and $\delta$ in the case of Clayton Copula. Par2 exists only for Student-t copula i.e. the degree-of-freedom $\nu$. The values between brackets give the standard error of the estimated parameters. Kendall's tau is a measure of rank correlation. Lower TD (Upper TD) is the lower (upper) tail dependence coefficient calculated from the copula parameters as shown in section 2.c..*

The shape parameters ($\kappa$) are all smaller than 2, meaning that the distributions of the residuals have fatter tails than the normal distribution. This suggests that extreme shocks are more likely to appear than in the case of Gaussian assumption.

### ii. Copula estimation results

Next, we estimate the copula parameters. Remind that we select the best specification from the three presented copulas. Table 4 reports the estimation results of the copula providing the best fit for the dependence structure in each of the ten market pairs.

The Student copula is the most preferred specification (in 6 out of 10 cases), followed by Gaussian copula (3 cases) and Clayton copula (one case).

In addition to the estimated parameters and their standard errors, three dependence coefficients are also represented: the Kendall's tau coefficient measuring the overall dependence and the lower (respectively upper) tail dependence coefficient (noted as LTD and UTD respectively) quantifying the relationship between extreme negative (respectively positive) events. In the next paragraphs, we are going to underline the results for some of the most interesting cross-asset pairs.

The dependence structure between the equities in the Developed Market (DM Eq.) and the euro zone sovereign bonds (EUR Sov.) is modeled by a Gaussian copula. Both the correlation parameter ($\rho=0.0475$) and the Kendall's tau ($\tau=0.0310$) suggest positive co-movement between the two assets. This implies that unlike U.S. Treasury bonds, EUR Sov. cannot be considered as safe havens against DM Eq. during times of distress.
This is not surprising considering that the EUR Sov. could be exposed to credit risk especially when the European sovereign debt crisis is included in our sample period. On the other hand, there is a negative relationship between the emerging market equities (EM Eq.) and EUR Sov., as indicated by $\rho=-0.0990$ and $\tau=-0.0663$. This pair is specified by a Student-t copula. The relatively small degree-of-freedom parameter ($\upsilon=9.2497$) indicates frequent extreme co-movements. LTD and UTD confirm the existing of this phenomenon between the two markets. These outcomes mark the potential use of EUR Sov. for hedging emerging market risk under normal market conditions, but this hedge becomes ineffective under extreme market conditions, as witnessed in the stressful episode over European government bond market since 2009.

In regard to the two equity markets, the links between EM Eq. and DM Eq. are described by a Gaussian copula with positive correlation parameter ($\rho=0.2632$).

During the past ten years, investors have experienced small cross region diversification benefits, which is the result of the globalization of economies and financial markets. Indeed, the interaction between developed economies and developing economies has been strengthened, due to free trade and activity offshoring. China's economy does suffer from the recession in the United States and Europe since 2008, and China's recent economic downturn is also seen to weigh on developed economies.

Overall, the globalization of the financial industry and the lowering of barriers to capital flows contributed to the reduction of cross-regional diversification opportunities. Regional specific shocks can quickly spill over to the global market.

Commodities are traditionally considered as attractive assets providing diversification benefits thanks to their weak and even negative correlation with equities. Nonetheless, as observed in our sample period, the interdependency between Commodity and EM Eq. is characterized by a Student-t Copula with a relatively large and positive correlation $\rho=0.3562$.

Commodity market seems to provide less diversification benefits in recent years. Additionally, LTD and UTD parameters suggest that commodities are not immune to tail events from emerging markets. Emerging economies are active actors on commodity markets. The demand for commodities from emerging countries is largely impacted by their activities, which are reflected by equity market performance.

As suppliers on commodity market, some other emerging economies' performances are also largely related to this market. The after-crisis recession reduces the commodity demand while the latter causes a self-enforcing negative effect on EM Eq.. Moreover, in a deleveraging and de-risking climate, investors sell off risky assets such as commodities and EM equities, contributing to their positive correlation.

Concerning the Foreign Exchange (FX) rate in our study (i.e. EUR-USD), an improving performance of this exchange rate reflects a strong Euro and a weak USD, and vice-versa. Accordingly, the positive correlations between FX and EM Eq. (Student-t copula with $\rho=0.1538$) as well as that between FX and Commodity (Gaussian copula with $\rho=0.3518$) actually correspond to negative correlations between USD and each of these two markets. In fact, a strong USD goes with an outflow of capital from EM stocks, leading an emerging market crash, especially in extreme cases (as demonstrated by LTD and UTD). Otherwise, commodities are priced in USD, which naturally justify their negative dependency.

Overall, from the estimated dependence structure over the past ten years, one could observe integrations not only for the same asset between different regions (as shown by EM Eq. and DM Eq.) but also for different asset classes. This could be even more visible under extreme market conditions as demonstrated by tail dependences. Diversification opportunities are less approachable; the "free lunch" is no longer on the table.

## 5. CONCLUDING REMARKS

In the first part of this note, we introduced the GARCH-Copula framework in the modeling of market returns with a special focus on extreme co-movements. The subsequent empirical study shows that the diversification benefits in multi-asset portfolio could be seriously diminished due to the intensified dependences between asset classes and between regions, especially under extreme market conditions. This is the result of macroeconomic instability in the context of the large integration of cross-regional and cross-asset markets and the globalization of economies.

In the next part of this note, we will give the simulation method used in our framework and demonstrate the calculation of risk indicators for cross-asset portfolios.

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