AWALEE NOTES

Copula in dependence modeling and risk measure estimating for cross-asset portfolio

Part II : Simulation and Application

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Table of contents

Summary: In this note (Part II), we first present the simulation process of returns in our GARCH-Copula approach. Using these Monte-Carlo (MC) type simulations, we formalize the estimation of two commonly used risk measures: VaR and Expected Shortfall (ES). We analyze the risk levels for chosen equally weighted portfolios over different horizons. The MC method gives closer risk estimations with respect to historical approach over the less stressed period. Depending on circumstances, the model specification is crucial for delivering appropriate risk indicators.

AWALEE NOTES

1. INTRODUCTION

Following the note "Copula in dependence modeling and risk measure estimating for cross-asset portfolio - Part I: Model and Estimation", we describe in this Part II the simulation process and its applications in estimating risk measures.

The measurement of risk is the core of risk management. Risk can be completely measured by probability distribution (if exists), which describes all the possible outcomes and their probabilities of occurrence. Nevertheless, it is simpler to describe the risk with one indicator.

Commonly adopted by financial markets practitioners and regulators, the Value-at-Risk (VaR) gives us the loss level that will not be exceeded with a confidence level during a period. VaR has impressive advantages like easy understandability, wide applicability and universality. In the work of Artzner, et al. in 1999, they formalize some desirable properties for a *coherent risk measure*. Nonetheless, VaR do not fullfill one of these properties: sub-additivity, which means that a portfolio merged from sub-portfolios should have a risk amount not greater than the sum of the risk amounts of the sub-portfolios. This actually indicates that diversification should reduce risk.

Since then, Expected Shortfall (ES), which gives the expected loss given that the loss is beyond a certain level, has been considered as a natural coherent alternative to VaR. Indeed, ES satisfies all the conditions for a *coherent risk measure*. ES begins to remplace VaR in many institutions for risk management. More recently, the Basel Committee on Banking Supervision recommends remplacing VaR by ES for internal market risk models in the Fundamental Review of the Trading Book initiative (BCBS, 2013).

In this note, based on our GARCH-copula approach, we introduce the simulation process of returns (Section 2). By implementing this Monte-Carlo type simulation, we escape from the multivariate normality constraint, considering that the copula can construct scenarii from different types of dependance structure (other than Gaussian one). After the returns are simulated, we formalize the estimation of VaR and ES in section 3. Section 4 is dedicated to an empirical study of risk measures for some typical equally weighted portfolios. We also compare the results of our approach with the historical approach of risk measure estimating. Section 5 concluds.

2. SIMULATION PROCESS

In this section, we describe the two-step simulation process of returns.

a. Uniform simulation

In the first step, by using the estimated bivariate copula model (Gaussian, Student-t or Clayton), we simulate two samples of uniform variates $\hat{u}_{1,t+k}$ and $\hat{u}_{2,t+k}$ with the dependence structure being taken into account. Note that if we simulate returns for producing risk measures with a ten-period liquidity horizon, we will need to simulate $k=1,\dots,10$ uniform variates for each series.

b. Return simulation

In the second step, the uniform variates are transformed into standardized residuals $\hat{z}_{1,t+k}$ and $\hat{z}_{2,t+k}$ by inverting their empirical cumulative distribution function (cdf). Knowing the information about σ_t, ϵ_t, r_t, we can therefore forecast $\hat{\sigma}_{t+k}$, $\hat{\epsilon}_{t+k}$, $\hat{r}_{t+k}$. In the one period case (i.e. $k=1$) for index $i=1,2$, we have

$$\hat{\sigma}^2_{i,t+1} = \hat{\boldsymbol{\omega}}_{\mathbf{i}} + \hat{\boldsymbol{\alpha}}_{\mathbf{i}}\epsilon^2_{i,t} + \hat{\boldsymbol{\beta}}_{\mathbf{i}}\sigma^2_{i,t} \tag{1}$$

$$\hat{\epsilon}_{i,t+1} = \hat{\sigma}_{i,t+1}\hat{z}_{i,t+1} \tag{2}$$

$$\hat{r}_{i,t+1} = \hat{\boldsymbol{\mu}}_{\mathbf{i}} + \hat{\mathbf{a}}_{\mathbf{i}} r_{i,t} + \hat{\epsilon}_{i,t+1} \tag{3}$$

For the k periods case, we just need to iterate this process to simulate the returns $\hat{r}_{i,t+k}$.

3. RISK MEASURES

By following recent changes in regulatory requirement, we estimate in this section the 99% VaR and the 97.5% ES for a portfolio investing in two assets.

In order to estimate the risk measures in different circumstances, we estimate the GARCH-Copula model for two different periods from our dataset: the nearest one year period (i.e. 01 October 2015 – 30 September 2016) and the stressed one year period (i.e. 01 January 2008 – 31 December 2008).

We estimate the model for different periods, with the aim of taking into account period-specific dependence structures. Moreover, considering that we want to compute the risk measures based on the last observation (i.e. 30 September 2016), the information about σ_t, ϵ_t, r_t is therefore given by their last observed values in our dataset.

For each of the 10,000 scenarios, we simulate a 10-day path of log returns for each asset ($\hat{r}_{i,t+1}$, $\hat{r}_{i,t+2}$, ..., $\hat{r}_{i,t+10}$) as described in the previous section.

The 1-day-ahead log return is simply given by $\hat{r}_{i,t+1}$, whereas the 10-day-ahead log return is calculated as:

$$\hat{r}_{i,t+10:t} = \hat{r}_{i,t+1} + \hat{r}_{i,t+2} + \ldots + \hat{r}_{i,t+10} \tag{4}$$

Supposing the portfolio is fully invested with weights $w_1 + w_2 = 1$, the log return of the portfolio r_p is calculated from the log returns of assets (r_1, r_2) as follows:

$$r_p = log\left(w_1 e^{r_1} + w_2 e^{r_2}\right) \tag{5}$$

In our case, we consider the equally weighted portfolios with two assets, thus $w_1 = w_2 = 0.5$.

The common used analytical methods for estimating risk measures are limited in the multivariate normal framework. Our Monte Carlo simulation is built on GARCH-Copula model, with the advantage of taking into account the dynamics in the returns and in the conditional variances, as well as the flexible dependence structure which is not limited to the normal distribution (i.e. Gaussian copula). Our framework is flexible enough to simulate random scenarios from the joint distribution of different asset returns based on different choices of marginal distributions and various alternatives of dependence structures.

As presented above, with the simulated portfolio return series r_p, the VaR for a given confidence level α is just the quantile of the asset return loss distribution. Note that the loss distribution is given by $-r_p$. More formally,

$$VaR_\alpha = F^{-1}_{-r_p}(\alpha) \tag{6}$$

where $F^{-1}_{-r_p}$ denotes the inverse of the loss distribution. The ES of the portfolio at level α is defined as the conditional expectation of the loss given that the loss is beyond the VaR level.

$$ES_\alpha = E\left[-r_p \middle| -r_p > VaR_\alpha\right] \tag{7}$$

In our case, the confidence level α is given by 99% for the VaR while it is 97.5% for the ES.

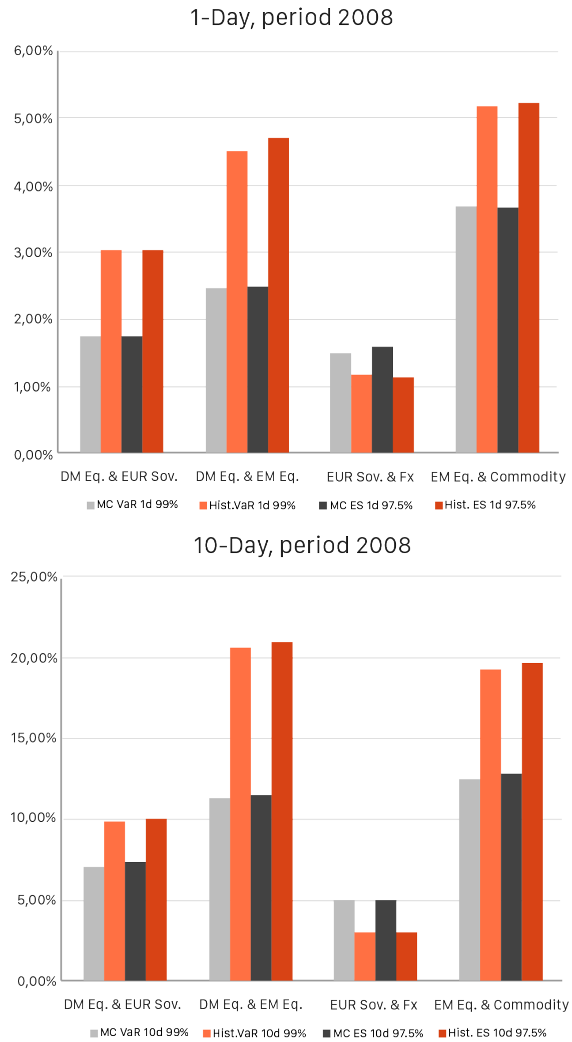
4. ESTIMATING PORTFOLIO RISKS

For the sake of simplicity, we consider several equally weighted portfolios. More precisely, we examine four portfolios as follows:

- Developed Market Equity (**DM Eq.**) and Euro Zone Sovereign bonds (**EUR Sov.**)
- **DM Eq.** and Emerging Market Equities (**EM Eq.**)
- **EUR Sov.** and Foreign Exchange EUR-USD (**FX**)
- **EM Eq.** and **Commodity**

We compute the two risk measures (VaR 99% and ES 97.5%) for these portfolios over two horizons (1 day and 10 days), by calibrating the model over two periods (2008 and 2015/2016). Moreover, in order to give a benchmark for our Monte-Carlo (MC) method, we also use the historical method to compute these risk measures.

Figure 1: Risk Measures, period 2008



Notes: The chart on the top shows the 1-day 99% VaR and the 1-day 97.5% ES based on two approaches (Monte-Carlo and Historical) for 2008 (stressed period). The chart on the bottom shows the same risk measures for a 10-day horizon. Note that the scales are different for the two graphs since the 10-day risk measures are consistently larger than the 1-day ones.

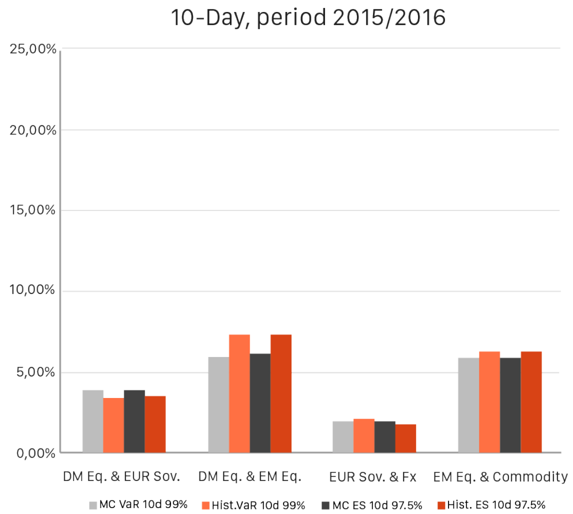
Figure 1 shows that the **EM Eq.** & **Commodity** portfolio and the **DM Eq.** & **EM Eq.** portfolio are the most risky ones since stock and commodity markets are more volatile than other markets.

The MC risk measures are smaller than those based on historical approach for three out of four portfolios. This could be explained by the simulation process used in our MC approach. Indeed, the initial conditions of our simulations are specified by the information of the last observation in our sample (i.e. 30 September 2016). These information (σ_t, ϵ_t, r_t) do not reflect the stressed situation in 2008 even if the stressed dependence structure has already been taken into account. If we use observations in stressed period for initial conditions, our MC method could give more accurate estimations of risk measures.

The 99% VaR and the 97.5% ES are rather close to each other in both approaches, with the later one slightly larger than the former one in 2008. This could justify the choice of 97.5% as the confidence level for ES considering his level is close to the VaR 99%.

If we look at the risk level between the two horizons, the **EM Eq.** & **Commodity** portfolio is riskier than the **DM Eq.** & **EM Eq.** for 1-day horizon, which is inversed in the case of 10-day horizon. This indicates that the horizon can change the relative riskiness between portfolios.

In order to scale the daily risk measure to the T-day one, one can use roughly the rule of multiplying the daily ones by the square root of T. Take the DM Eq. & EM Eq. portfolio for example, the historical one day 99% VaR is 4.51%, multiplying by $\sqrt{10}$ gives us a 10-day VaR of 14.26%. However, the real historical 10-day 99% VaR is 20.63%. In our case, multiplying the 1-day measure by $\sqrt{T}$ could largely underestimate the 10-day risk level, especially over stressful period.



Notes: The first sub-chart shows the 1-day 99% VaR and the 1-day 97.5% ES based on two approaches (Monte-Carlo and Historical) for 2015/2016 (the most recent period). The second one shows the same risk measures for a 10-day horizon. Note that the scales are different between the two. However, the scales of the corresponding graph are kept the same between Figure 1 and Figure 2 to better compare the risk measures between two periods.

Figure 2 presents the risk measures for the most recent one year period (2015/2016). Comparing to the stressed period (2008) in Figure 1, one can notice that the risk measures are much smaller in the 2015/2016 period, which is highlighted by the two estimation methods. This indicates that the recent period is less stressful than 2008. The 99% VaR remains close to the 97.5% ES in both approaches.

Different from the stressed period, we note that in the most recent period, the MC method gives lightly larger risk levels than the historical method. Considering that our initial conditions are collected from the end of this period, MC method gives relatively close risk estimations to the historical benchmark estimations.

Figure 2: Risk Measures, period 2015/2016

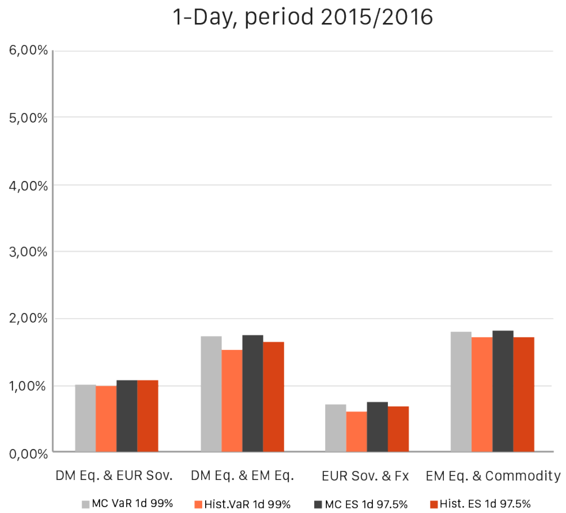
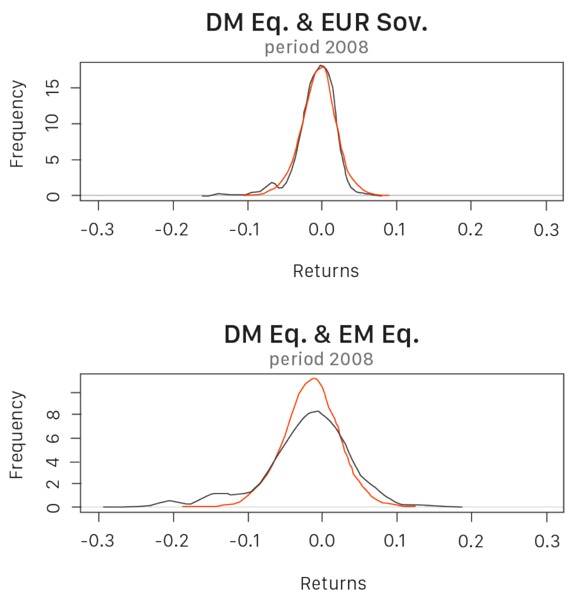
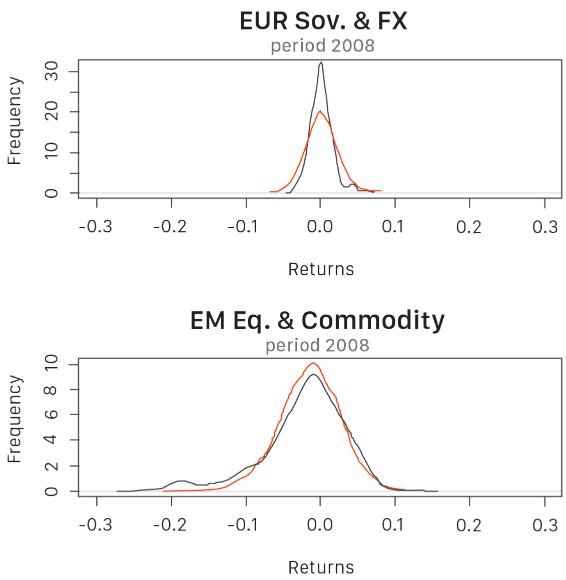


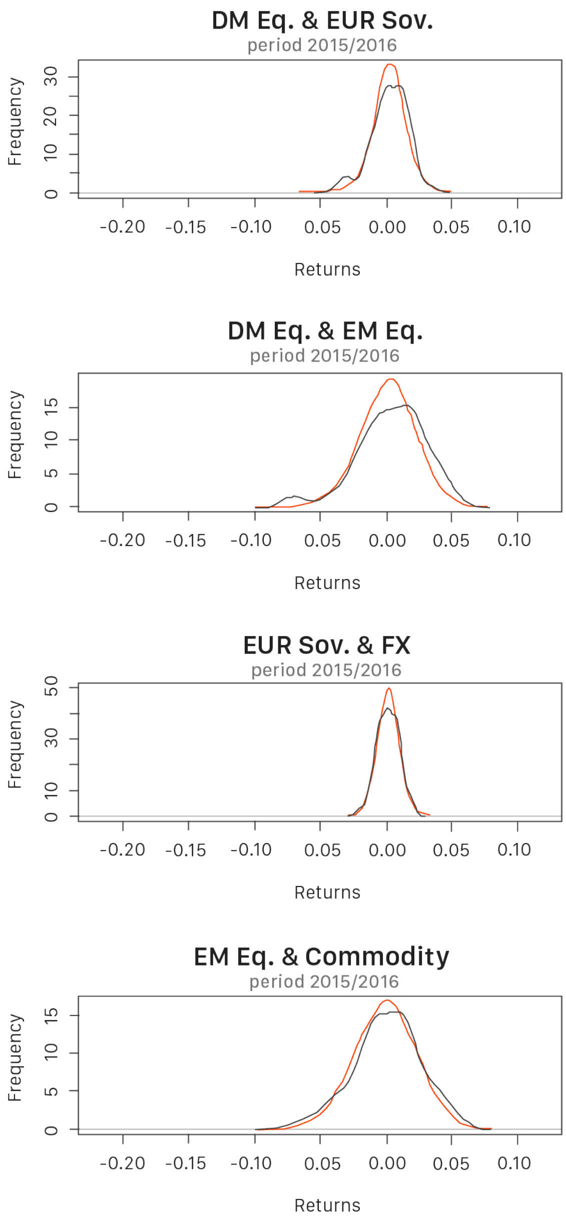
Figure 3: Density of returns (Period 2008)





Notes: This figure shows the density function of the 10-day returns for the 4 portfolios, based on two approaches: MC (Red line) and Historical (Black line), over the stressed period (2008).

Figure 4: Density of returns (Period 2015/2016)



Notes: This figure shows the density function of the 10-day returns for the 4 portfolios, based on two approaches: MC (Red line) and Historical (Black line), over the most recent period (2015/2016).

Figure 3 and 4 exhibit the density distributions of simulated returns and historical returns for the four portfolios over the two periods. Globally, the two methods are closer for the less stressed period comparing to the period 2008 for the reason explained above.

The **DM Eq.** & **EUR Sov.** portfolio has a left fatter tail in the historical approach which is not really captured by MC simulation. The two riskier portfolios (**DM Eq.** & **EM Eq.** and **EM Eq.** & **Commodity**) have a distribution with bigger probability around the mean and thinner tails in the MC method with respect to historical method. On the other hand, the **EUR Sov.** & **FX** portfolio has a more clustered distribution in the historical method. These deviations indicate that the model needs to be better calibrated portfolio by portfolio, especially in terms of initial conditions and marginal distributions.

5. CONCLUSION

In this part II, we presented the simulation process of returns in our GARCH-Copula approach and the related risk measures estimation. By re-estimating the model over two selected periods, we study the risk levels for the chosen portfolios over different horizons. We compare results from the MC method to those from historical approach. The MC method gives closer risk estimations with respect to historical approach over the less stressed period since the initial conditions for the simulation are specified from this period.

However, we are aware that the model should be calibrated case by case in terms of specifications in order to obtain more appropriate risk measure estimation. A more complete application of copula approach is given in Bruneau, et al., (2015) who developed a non-linear risk factor model for large cross asset portfolio risk measuring.

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