AWALEE NOTES

The Free Boundary SABR Model

Study carried out by the Quantitative Practice,
special thanks to Mehdi Aithmidou

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FEVRIER 2018

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1. INTRODUCTION

This study aims at developing a stochastic volatility model in order to determine the value of interest rate derivatives under the specific framework of negative rates. The objective of this paper is to show an interesting extension to the current volatility models used by market operators. In the following, we will take the case of swaption pricing as an example, that is a derivative product that consists in an option contract giving the right to enter into an underlying swap. We will study the necessity to develop such a model in the current financial environment. To this end, we will introduce the Free Boundary SABR model.

With the latest changes following the 2007 crisis, financial institutions are facing more challenges with an increased level of complexity. Among these, particularly noteworthy is the challenge of negative interest rates. In order to stimulate the economy, Central Banks have decided to lower the level of interest rates. Indeed, a lower level of interest means a lower cost of credit, agents have then more incentive to borrow money and use it to invest. Traditionally, a level of interest was considered to be equal to zero at the lowest. In the context of global financial crisis, keeping interest rates close (positive) or equal to zero can prove insufficient to stimulate investment. Lowering rates to negative values would then have a more efficient impact (money deposits become costful).

Negative interest rates levels have major consequences for the valuation of interest rate derivatives since their value depends on these underlying rates. As a consequence, volatility modeling becomes more tricky while it is a fundamental step in derivatives pricing. The main issue for market operators is to adapt volatility quoted on the market to negative rates. In the next sections, we will present how to overcome this issue by using the Free Boundary SABR. To do so, we will show all the necessary steps to build this new modeling framework.
The critical purpose of this study is to develop a robust and fast interpolator for volatility cubes that are exhibited on the market interest rates. This interpolator must also include a limited number of parameters in order to easily interpret their impacts and understand their dynamics and also to make the calibration easier.

2. PREVIEW

2.1. Observations

Let's start with an observation taken from [3], by looking at the historical evolution of the Swiss Franc interest rates (CHF) see the following Figure (1):

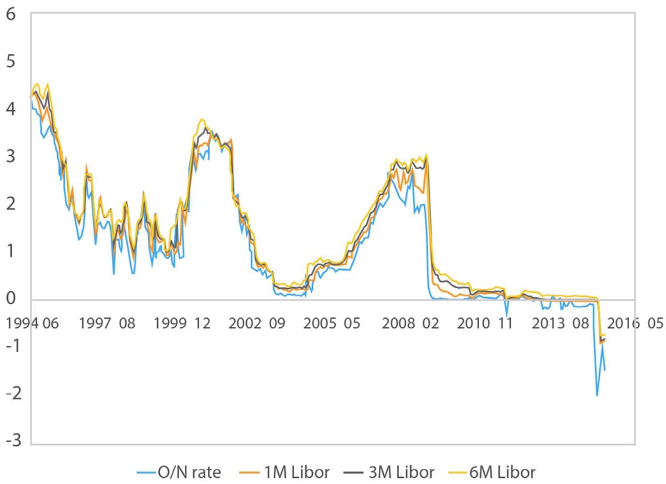

Figure 1: CHF: Swiss Franc interest rates

One could notice that, for instance, the Overnight interest rate (the blue curve) reached sometimes −2%. Moreover, the level of interest rates sticks to zero for a certain period of time, which mathematically means that their probability density should present more or less a singularity in zero.

2.2. Negative Rates Handling

The classical method used nowadays by every financial institution consists in shifting the SABR (see equation 1), which will move the lower bound of F_t from *0* to *-s*. This is what has been introduced in the shifted version of the Arbitrage-Free SABR (cf. Appendix.E of [1]):

$$\begin{cases} dF_t = \nu_t (F_t + s)^{\beta} dW_t^1 \\ d\nu_t = \gamma \nu_t dW_t^2 \\ \nu_0 = \alpha \\ < dW_t^1, dW_t^2 >= \rho dt \end{cases} \tag{1}$$

To do so, there are two alternatives. The first method consists in introducing the shift in the calibration parameters (α,β,ρ,γ,s), i.e. calibrate s as a parameter of the model as the other SABR parameters by using Levenberg-Marqualt algorithm on (α, β, ρ, γ, s). The second method aims at fixing the shift to −2% if we work, for example, on the Swiss interest rates. However the two methods present drawbacks, indeed:
• The first method doesn't work as well as expected because the shift doesn't introduce any new degree of freedom, actually the shift influences the skew in the same way as the elasticity coefficient β.

• The second method doesn't work either, indeed, fixing manually the shift could have a great impact in the pricing and hedging, because the interest rates could potentially drop down lower than the shift. This will result in a jump in the calibration parameters, translated by a jump on the Greeks. Thus, it will have two major consequences: forcing the traders to reserve a hedging P&L part and constraining the product price to be bounded from above (the swaption price in our case). This can lead to unattainable market prices for the shifted SABR.

2.3. The Model

The Free Boundary SABR Model was introduced by Antonov (2015) and revisited in different versions of his paper (cf. [2] to [4]). Contrary to the standard SABR framework, this model aims at providing a natural extension to negative rates with the idea of deriving an exact solution for the zero correlation case and extending it to the general one with suitable approximations.

$$\begin{cases} dF_t = \nu_t |F_t|^{\beta} dW_t^1 \\ d\nu_t = \gamma \nu_t dW_t^2 \\ \nu_0 = \alpha \\ < dW_t^1, dW_t^2 >= \rho dt \end{cases} \quad \text{with } 0 \leq \beta < \frac{1}{2} \tag{2}$$

This model takes into account negative rates and presents a stickiness in *0* which is observed in the market (see Figure 1). This model also conserves the total probability (the norm) and the martingality (the first moment) which guarantees its arbitrage freeness. The β boundaries condition allows the forward rate to cross the zero point, indeed if we take $\beta > 1/2$, the forward rates movement will decrease rapidly when approaching the zero point without ever reaching it (as the famous Zeno's Paradox).

The idea of the derivation of the Free Boundary SABR comes from the current literature. It is actually based on a study regarding the probability density of the forward rate followed by a CEV process (see Brecher and Lindsay, 2010 [7]), as well as different studies about the call time value of a forward rate following this process (see P. Carr [5] and Antonov, Proposition 1 [6]). Moreover, if we consider a Free-CEV process then we also know its probability density (see Antonov 2015 [2]), which allows us to get the call time value for the Free-CEV with the same calculus as mentioned before. Furthermore, we can notice that the call time value of the Free-SABR zero correlation process could be written as the expected value of the call time value of the Free-CEV process (see the following equation 3). Since the call time value of the Free-CEV is a one-dimensional integral, the call time value of the Free-SABR zero correlation will be a two-dimensional integral.

$$O_F^{SABR_{\rho=0}}(T,K) = E\left[(F_T-K)^+\right] - (F_0-K)^+ = E\left[O_F^{CEV}(\tau_T,K)\right] \tag{3}$$

With the integrated variance $\tau_T = \int_0^T \nu_t^2 dt$. First, we will search for an exact solution of the call time value for the Free-SABR zero correlation process and extend this to the general case by using a mapping toward the zero correlation through the Free-SABR parameters.
To do so, we will use a heat kernel expansion (see De Witt 1965 [8]) which is an asymptotic approximation (in maturity) for the parabolic PDEs.

Before getting directly to the Free-SABR implementation, let's get some intuition about the CEV process and the Free-CEV process.

3. INTUITION ON THE CEV MODEL

3.1. The CEV Model

Let's consider the following CEV model:

$$dF_t = \nu F_t^{\beta} dW_t \qquad \text{with } 0 \leq \beta \leq 1 \tag{4}$$

The associated Forward Kolmogorov PDE is $p_t - \frac{1}{2}(\nu^2 f^{2\beta} p)_{ff} = 0$ with p the probability density associated to F. One notices that fixing the SDE alone doesn't give a unique solution. Indeed, there exists two main solutions, the absorbing one and the reflecting one. Thus, we must add a boundary condition on zero to uniquely define the solution.

Moreover, as it was mentioned before, one needs to choose $\beta < 1/2$ to get the existence of the reflecting solution (see Feller Classification [9]).

According to the literature (see [7] or p.4 of [2]), if we choose $\nu = 1$ for simplification, the absorbing and reflecting densities are defined as follows:

$$\begin{cases} p_A(t,f) = \frac{1}{1-\beta} \frac{f^{1-2\beta}}{t} \left(\frac{f}{f_0}\right)^{-\frac{1}{2}} e^{-\frac{q^2+q_0^2}{2t}} I_{|\nu|}\left(\frac{q q_0}{t}\right) \\ p_R(t,f) = \frac{1}{1-\beta} \frac{f^{1-2\beta}}{t} \left(\frac{f}{f_0}\right)^{-\frac{1}{2}} e^{-\frac{q^2+q_0^2}{2t}} I_{\nu}\left(\frac{q q_0}{t}\right) \end{cases} \tag{5}$$

For $\nu = -\frac{1}{2(1-\beta)}$, $F_0 = f_0$ with $q_0 = \frac{f_0^{1-\beta}}{1-\beta}$, $q = \frac{f^{1-\beta}}{1-\beta}$ and $I_\nu(x)$ is the modified Bessel function of order ν.

Thus, the absorbing and reflecting asymptotic densities at zero are:

$$P_A \sim_0 f^{1-2\beta} \qquad P_R \sim_0 f^{-2\beta}$$

In order to study the arbitrage freeness, we calculate the conservation of the norm $\frac{\partial_t}{\partial t}\left(\int_0^{+\infty} p(t,f)df\right)$ and of the first moment (martingality) $\frac{\partial_t}{\partial t}\left(\int_0^{+\infty} fp(t,f)df\right)$.

To do this, one should use an integration by parts relying on the Fokker-Planck equation.

The results are in the following table 3.1:

	Absorbing Solution	Reflecting Solution
Norm Conservation	No	Yes
First Moment Conservation	Yes	No

Table 1: Summary of properties of absorbing and reflecting solutions

We notice that the reflecting solution preserves the total probability but not the martingality and the absorbing solution preserves the martingality but not the total probability. This means that, in order to have a correct financial process, we must keep a part of every solution and that is why we must choose $\beta \leq 1/2$ in the first place.

3.2. The Free-CEV Model

$$dF_t = \nu |F_t^\beta| dW_t \qquad with\ 0 \leq \beta \leq \frac{1}{2} \tag{6}$$

According to [2], the solution of the Fokker-Planck equation associated to the PDE above (6) that satisfies the initial condition $p(0,f)=\delta(f-f_0)$ and guarantees the norm and first moment preservation could be written explicitly in terms of absorbing and reflecting solution for the CEV Model, as:

$$p(t,f) = \frac{1}{2}\left(p_R(t,|f|) + sign(f)p_A(t,|f|)\right) \tag{7}$$

3.3. Numerical Results

In the following experiment, we will show the behavior of the Free-CEV, the absorbing solution and the reflecting one; moreover, we will note the β effect on those solutions around the critical point zero. Let's start with the following experiment parameters.

Parameter	Symbol	Value
Rate Initial Value	F_0	50 bps
SV Initial Value	ν_0	$0.6\ F_0^{1-\beta}$
Skews	β	0.1 and 0.25
Maturities	T	$3Y$

Table 2: Setups for the Free-CEV Tests

On the following figures below (see 2 and 3), we notice the spike presented by the Free-CEV, which represents in a way the stickiness of rates at zero in the market as observed in the Figure (1). We know that the density around the critical point zero is divergent with $p(t,f) \sim_0 f^{-2\beta}$ which is the result of the presented spike, and to be more precise, it is a spike and not a delta singularity, according to the Riemann integral convergence condition on zero and due to $-2\beta > -1$.

Another point is to remind that the absorbing density of probability has an asymptote in zero of $f^{1-2\beta}$ which means that it doesn't diverge at all.
Furthermore, as we already said, the absorbing solution doesn't conserve the norm, so if we have to work with it alone, we should add a delta in zero to conserve it, which is not the case here, since we also have the reflecting solution part.
Moreover, the reflecting solution has an asymptote at zero of $f^{-2\beta}$ which means that it is responsible for the spike in the Free-CEV process at zero.

Finally, we notice in the figures below (see 4 and 5) that by increasing β from 0.1 to 0.25, we get a bigger spike at zero which is expected by the asymptotic study.

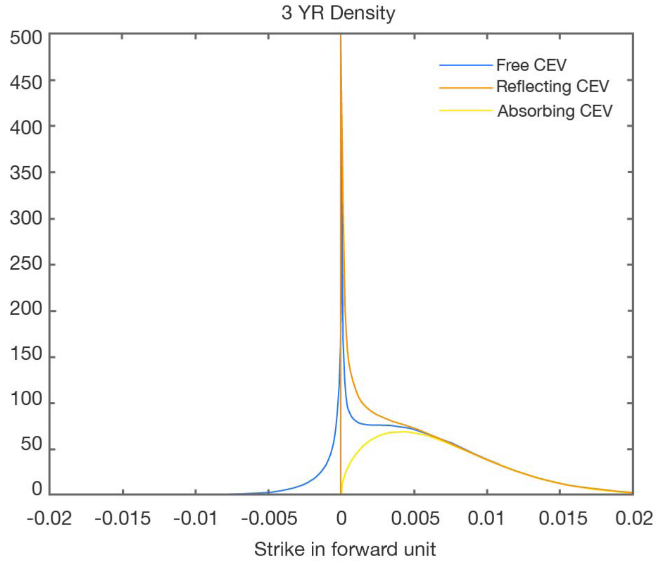

Figure 2: $\beta = 0.25$

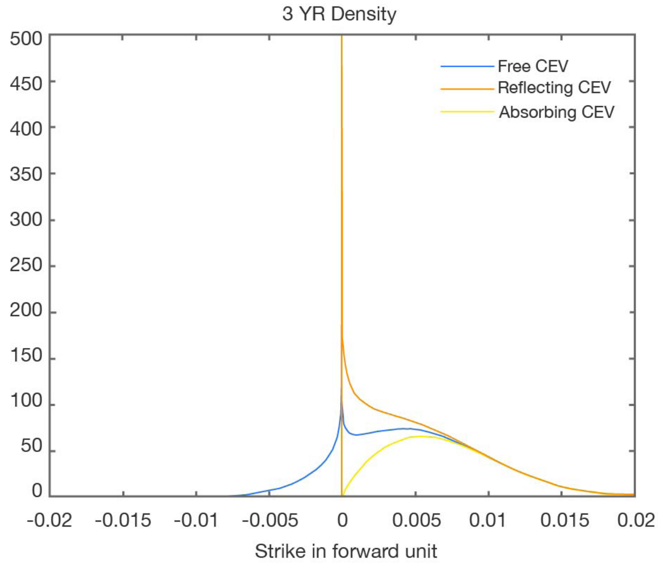

Figure 3: $\beta = 0.1$

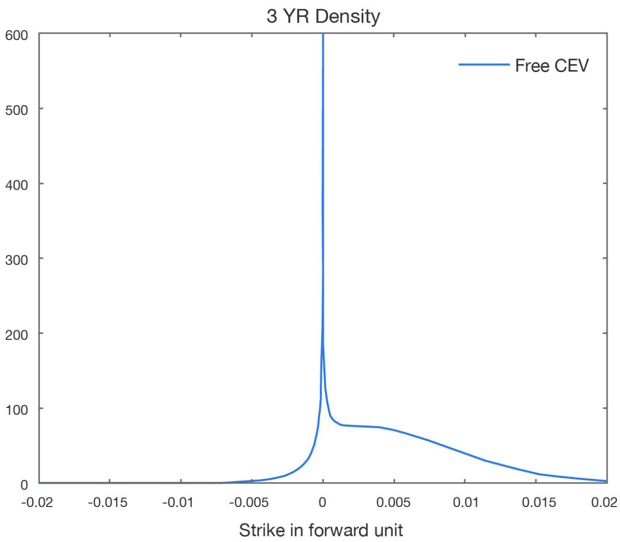

Figure 4: $\beta = 0.25$

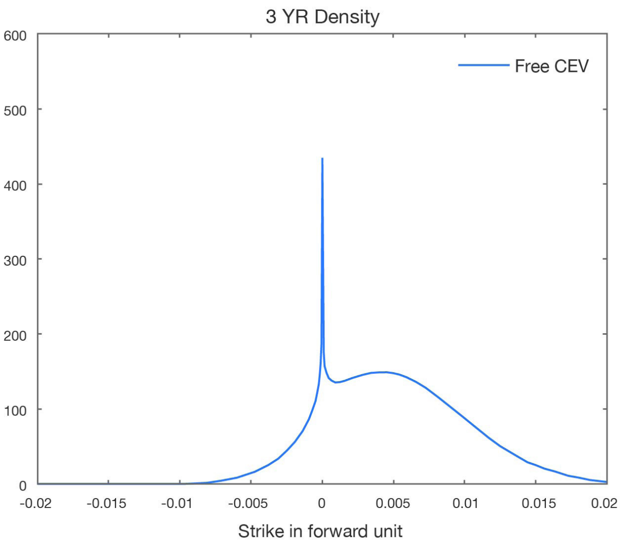

Figure 5: $\beta = 0.1$

4.IMPLEMENTATION OF THE FREE SABR

Let's consider the following set of parameters:

Parameter	Symbol	Value
Rate Initial Value	F_0	50 bps
SV Initial Value	ν_0	$0.6\ F_0^{1-\beta}$
Vol-of-Vol	γ	0.3
Correlations	ρ	-0.3
Skews	β	0.1 and 0.25
Maturities	T	3Y

Table 3: Setups for the Free-CEV Tests

4.1. The Zero Correlation Case

In the zero correlation case of the Free Boundary SABR, the call time value is given by:

$$O_F^{SABR}(T,K) = \frac{1}{\pi}\sqrt{|KF_0|}\left\{1_{K\geqslant 0}A_1 + \sin(|\nu|\pi)A_2\right\} \qquad (8)$$

With

$$\begin{cases} A_1 = \int_0^{\pi} \frac{\sin(\phi)sin(|\nu|\phi)}{b-\cos(\phi)} \frac{G(T\gamma^2, s(\phi))}{\cosh(s(\phi))} d\phi \\ A_2 = \int_0^{+\infty} \frac{\sinh(\psi)(1_{K\geqslant 0}\cosh(|\nu|\psi)+1_{K<0}sinh(|\nu|\psi))}{b+\cosh(\psi)} \frac{G(T\gamma^2, s(\psi))}{\cosh(s(\psi))} d\psi \end{cases} \qquad (9)$$

$$G(t,s) = 2\sqrt{(2)}\frac{e^{-\frac{t}{8}}}{t\sqrt{2\pi t}}\int_s^{+\infty} ue^{-\frac{u^2}{2t}}\sqrt{\cosh(u)-\cosh(s)}du$$

(10)

And

$$\begin{cases} \sinh(s(\phi)) = \gamma\nu_0^{-1}\sqrt{2\bar{q}(b-\cos(\phi))} \\ \sinh(s(\psi)) = \gamma\nu_0^{-1}\sqrt{2\bar{q}(b+\cosh(\psi))} \end{cases} \qquad \begin{cases} \bar{q} = \frac{|f_0 f|^{1-\beta}}{(1-\beta)^2} \qquad \nu = \frac{-1}{2(1-\beta)} \\ b = \frac{|f_0|^{2(1-\beta)}+|f|^{2(1-\beta)}}{2|f_0 f|^{1-\beta}} \end{cases}$$

Since all the simulations are done with Matlab, we have come across a snag, because Matlab doesn't have a functionality that calculates directly double integrals (see equation 8) with variable bounds (here is s, that also varies in terms of the strike). So, in order to implement this double integral, we started by implementing a one-dimensional integral nested in another one-dimensional integral. But first we implemented the integral $(t,s) \rightarrow G(t,s)$ by using the indicator function $1_{u>s}$ to fix the bounds and a Gauss-Hermit because of the weight of e^{-u2} in the integral.

However, this quadrature didn't work, we found out that it was due to the presence of the cosh(u) term. Indeed, this first implementation doesn't answer correctly the check of having $G(t,0) = 1$ for every t. This last result comes from the fact that:

$$G(t,s) = 2\pi\int_s^{+\infty} G_{McKeanKernel}(t,u)sinh(u)du = P(d(x,y) > s)$$

With d the hyperbolic distance in H^2 (the hyperbolic Poincaré plane) and $G_{McKeanKernel}$ is the McKean Kernel function which preserves the norm.

For this reason, we used a Gauss-Legendre Quadrature, by having previously used a $u \rightarrow arctan(u)$ variable change to reduce the integral to a bounded interval. Even so, we still got instabilities in cumulative and density calculus, this comes from the fact that the Gauss-Legendre discretization grid points subtly changed with the strike.

In fact, this was due to the use of an indicator function $1_{u>s}$ in our calculus that removed or added some Gauss-Legendre grid points every time we changed the strike (it changed directly the s variable). Finally, in order to resolve this last matter, we used a $u \rightarrow u - s$ variable change and the appropriate trigonometric formula to simplify $cosh(u) - cosh(s)$.

4.2. The General Case

Let's consider the general Free Boundary SABR process as defined in 11. The idea is to consider a new process F that is a Free Boundary with zero correlation, defined with the tilde parameters and satisfying:

$$\begin{cases} d\tilde{F}_t = \tilde{\nu}_t|\tilde{F}_t|^{\tilde{\beta}} d\tilde{W}_t^1 \\ d\tilde{\nu}_t = \tilde{\gamma}\tilde{\nu}_t d\tilde{W}_t^2 \\ \tilde{\nu}_0 = \tilde{\alpha} \\ < d\tilde{W}_t^1, d\tilde{W}_t^2 >= 0 \\ E\left[(F_t - K)^+\right] \approx E\left[(\tilde{F}_t - K)^+\right] \end{cases} \quad \text{with } 0 \leq \tilde{\beta} < \frac{1}{2} \qquad (11)$$

With the Heat Kernel Expansion (see Paulot 2009 [10] or Labordère 2008 [11]) we get the following mapping:

$$\tilde{\beta} = \beta \qquad \tilde{\gamma}^2 = \gamma^2 - \frac{3}{2}\left\{\gamma^2\rho^2 + \nu_0\gamma\rho(1-\beta)F_0^{\beta-1}\right\}$$

$$\tilde{\nu}_0 = \tilde{\nu}_0^{(0)} + T\tilde{\nu}_0^{(1)} \qquad \tilde{\nu}_0^{(0)} = \frac{2\Psi\delta\tilde{q}\tilde{\gamma}}{\Psi^2 - 1}$$

$$\Psi = \left(\frac{\nu_{min} + \rho\nu_0 + \gamma\delta q}{(1+\rho)\nu_0}\right)^{\frac{\tilde{\gamma}}{\gamma}} \qquad \nu_{min}^2 = \gamma^2\delta q^2 + 2\gamma\rho\delta q\nu_0 + \nu_0^2$$

$$\delta q = \frac{|k|^{1-\beta} - |F_0|^{1-\beta}}{1-\beta} \quad \text{and} \quad \delta\tilde{q} = \frac{|k|^{1-\tilde{\beta}} - |F_0|^{1-\tilde{\beta}}}{1-\tilde{\beta}}$$

$$k = max(K, 0.1F_0) \approx 0.1F + \frac{1}{2}\left(K - 0.1F + \sqrt{(K-0.1F)^2 + \epsilon^2}\right)$$

$$\epsilon = 1bps,$$

in order to avoid non-smooth behavior around $F_0 = 10K$

$$\frac{\tilde{\nu}_0^1}{\tilde{\nu}_0^0} = \tilde{\gamma}^2\sqrt{1+\tilde{R}^2}\,\frac{\frac{1}{2}\ln\left(\left|\frac{\nu_0\nu_{min}}{\tilde{\nu}_0^{(0)}\tilde{\nu}_{min}}\right|\right) - B_{min}}{\tilde{R}\ln\left(\sqrt{1+\tilde{R}^2}+\tilde{R}\right)} \qquad \tilde{R} = \frac{\delta q\tilde{\gamma}}{\tilde{\nu}_0^{(0)}} \qquad \tilde{\nu}_{min} = \sqrt{\tilde{\gamma}^2\delta q^2 + \left(\tilde{\nu}_0^{(0)}\right)^2}$$

$$B_{min} = -\frac{1}{2}\frac{\beta}{1-\beta}\frac{\rho}{\sqrt{(1-\rho^2)}}(\pi - \phi_0 - \arccos(\rho) - I)$$

$$L = \frac{\nu_{min}}{q\gamma\sqrt{1-\rho^2}} \qquad \phi_0 = \arccos\left(-\frac{\delta q\gamma + \nu_0\rho}{\nu_{min}}\right)$$

$$I = \begin{cases} \frac{2}{\sqrt{1-L^2}}\left(\arctan\left(\frac{u_0+L}{\sqrt{1-L^2}}\right) - \arctan\left(\frac{L}{\sqrt{1-L^2}}\right)\right) & \text{for } L < 1 \\ \frac{1}{\sqrt{L^2-1}}\ln\left(\frac{u_0(L+\sqrt{L^2-1})+1}{u_0(L-\sqrt{L^2-1})+1}\right) & \text{for } L > 1 \end{cases}$$

$$u_0 = \frac{\delta q\gamma\rho + \nu_0 - \nu_{min}}{\delta q\gamma\sqrt{1-\rho^2}} \qquad q = \frac{k^{1-\beta}}{1-\beta}$$

• A special attention should be given to the ATM case. Indeed, in this case, we have to put:

$$\begin{cases} \tilde{\nu}_0^{(0)} = \nu_0 \\ \frac{\tilde{\nu}_0^{(1)}}{\tilde{\nu}_0^{(0)}} = \frac{1}{12}\left(1 - \frac{\tilde{\gamma}^2}{\gamma^2} - \frac{3}{2}\rho^2\right)\gamma^2 + \frac{1}{4}\beta\rho\nu_0\gamma F_0^{\beta-1} \end{cases} \qquad (12)$$

4.3. Numerical Results

By considering the following set of parameters:

Parameter	Symbol	Value
Rate Initial Value	F_0	50 bps
SV Initial Value	ν_0	$0.6\ F_0^{1-\beta}$
Vol-of-Vol	γ	0.3
Correlations	ρ	-0.3
Skews	β	0.25
Maturities	T	3Y

Table 4: Setups for the Free-CEV Tests

We get the following results (see figures 6, 7 and 8) with the general Free Boundary quadrature algorithm implementation.

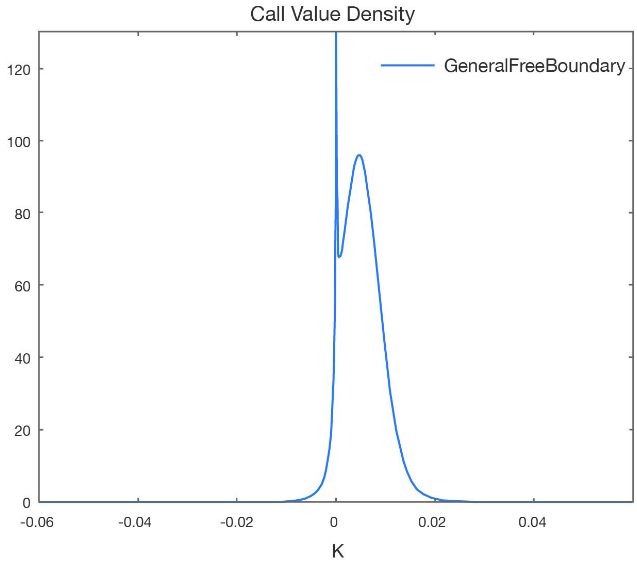

Figure 6: Density of the Analytical Free Boundary SABR

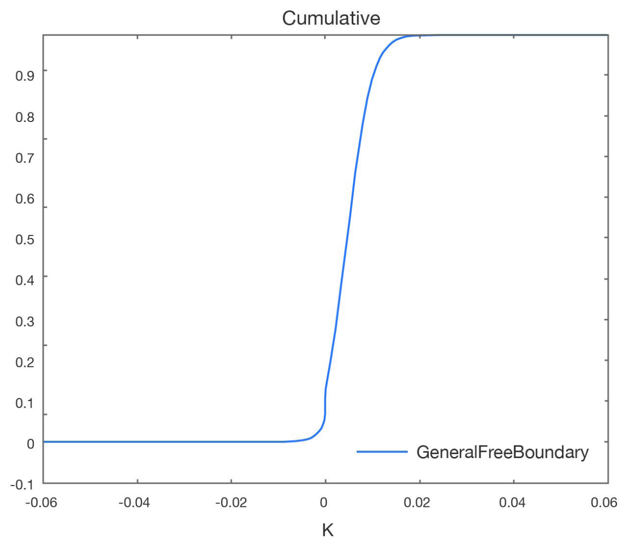

Figure 7: Cumulative of the Analytical Free Boundary SABR

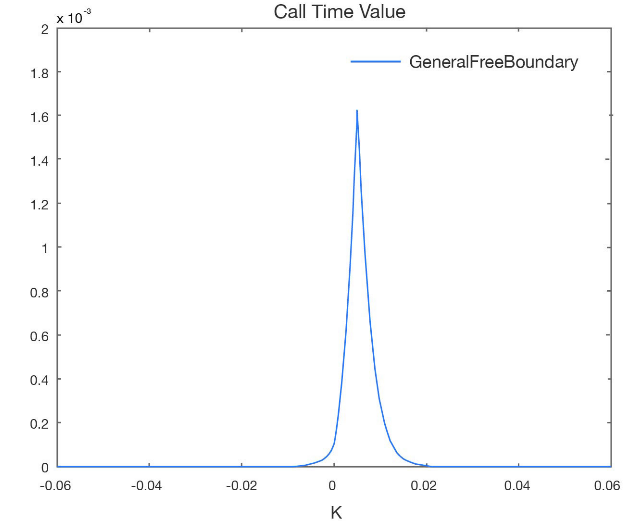

Figure 8: Call Time Value of the Analytical Free Boundary SABR

And finally, to confirm the shape of the analytical result, here is a Quasi Monte-Carlo simulation using Sobol sequences (see figure 9), with N = 512 space steps and M = 100,000 paths, done with simulating (with an Euler scheme) the Bessel squared process X_t.

$$X_t = sign(F_t)\frac{|F_t|^{2(1-\beta)}}{(1-\beta)^2} \tag{13}$$

So

$$dX_t = 2sign(X_t)(v+1)\nu_t^2 dt + 2\nu_t\sqrt{X_t}dW_t \text{ where } v = -\frac{1}{2(1-\beta)} \tag{14}$$

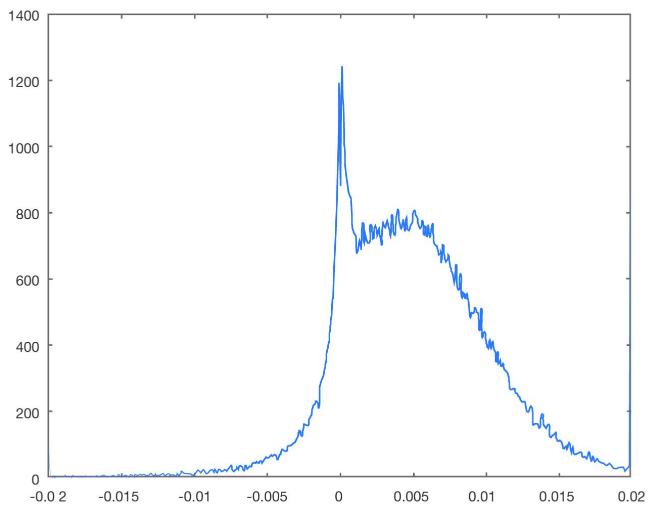

Figure 9: Density of the Quasi Monte Carlo Free Boundary SABR

5. CONCLUSION

We could notice from this study of the Free Boundary SABR several observations:

The first point to notice is the stickiness in zero which was expected from the Free Boundary SABR Model, since this represents one of the market observations.
Secondly, we see that the forward rate has many patterns, one that could go to the high values with low probabilities, another one is to stick to the spot with a high probability. On the other hand, if the forward rate goes left, there are two cases, either it will stick in zero with spike of probability or cross this zero point to negative rates with low probabilities because of the thick tail.

The delta of the Free Boundary SABR is smooth everywhere.

The Gamma of the Free Boundary is smooth everywhere, except on zero, indeed: $\frac{\partial^2 Call_{FreeBoundarySABR}}{\partial F^2} \sim_0 |F_0|^{-2\beta}$, however we could avoid this inconvenience by using a finite difference method with steps of about 1 to 5 bps.

At the opposite of the Free Boundary that presents a zero skew of implicit volatility in zero (strike), the Free Boundary SABR Model can control this skew thanks to its correlation coefficient ρ.

The Free Boundary SABR is a quite good model but its arbitrage freeness could be questioned for very long dated options (for example 30 years). In this case, the mapping for doing the expansion doesn't hold anymore, which means that this model is near arbitrage free.

For this reason, A. Antonov recently introduced a new model: the Mixture SABR ([12]). It is a weighted sum of the Normal Free Boundary SABR and the zero correlation Free Boundary SABR, adding more degrees of freedom without losing in computation time because the Normal Free Boundary SABR adds just one-dimensional integral to be calculated. This last model is Arbitrage Free by construction since it is a weighted sum of two Arbitrage Free Models.

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